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Study of nonlinearity and filamentation of ultra-short laser pulses in plasmas

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SRIVIPRA PROJECT 2022

Title: Study of Relativistic ponderomotive nonlinearity and filamentation of ultra-short laser pulses in plasmas

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Certificate

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project SVP-2235 titled **“Study of Nonlinearity & Filamentation of Ultra-Short Laser Pulses in Plasmas”**. The participants have carried out the research project work under my guidance and supervision from …21st June 2022 to 25th September 2022. The work carried out is original and carried out in an online mode.

Signature of Mentor

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ABSTRACT

We study the setting up of relativistic ponderomotive nonlinearity in under-dense collisionless plasma. Using the fluid model, coupled system of equations of laser beam and electron plasma wave has been derived. We present the numerical simulation for this coupled system of equations, when the coupling arises through relativistic ponderomotive non-linearity. The results show that with time, localized structures become more complex and the plasma oscillations frequency spectra have several harmonics peaks at terahertz frequencies when the electron plasma frequency is in terahertz range and laser frequency is around 2.4 $x10^{15}$ sec^{-1}. We also present the semi analytical model to capture the underlying physics.

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I. INTRODUCTION

The laser-plasma techniques involves laser pulse energy absorption and its coupling with electrostatic modes of plasma [1-4]. In laser pump applications the nonlinear processes like resonant absorption, decay and modulation instability, self-focusing and filamentation etc. require deep understanding of nonlinear interaction of high power laser with plasma. The success of laser induced fusion application are hindered by the parametric instabilities[5-9]. The nonlinearity in laser applications arises due to the large quiver velocity of electrons gained due to the absorption of high energy from the lase pulse.

Interaction of high power laser with plasma is a gateway to many new frontier physics experiments. High intensity laser on plasma can create very high density matter, allowing us to study such states of matter in laboratories. Due to its important applications in laser-driven accelerators, laser fusion, ionospheric radio wave propagation, x-ray lasers, and other fields, electromagnetic beam interactions with plasmas have received a great deal of attention in experimental and theoretical studies. Since powerful laser beams need to propagate over several Rayleigh lengths in these applications, self-focusing and self-trapping electromagnetic beams hold special places in laser-plasma interactions.

Plasma contains free electrons and ions. When laser is made to propagate inside plasma, its electric field causes the free charges to move thereby generating a conduction current. This conduction current opposes the displacement current of the em radiation of laser. Hence the propagation of laser is inhibited inside plasma. But if the laser frequency is higher than the electron plasma frequency, laser can propagate inside plasma. Thus, there is a cutoff plasma density corresponding to the frequency above which laser cannot propagate inside it.

Underdense plasma is the plasma with density less than the critical plasma density, and laser propagation can only happen in underdense plasma.

Laser propagation inside plasma is strongly nonlinear. As laser propagates inside plasma, its energy is dissipated in collisions, i.e. the electromagnetic radiation is absorbed by electrons and other free charges and radiated as thermal and kinetic energy. However with ultra intense laser for intensity near 10^18 W/cm^2, relativistic nonlinear effects start appearing and effects of collisions and absorption diminish. The laser acts like a powerful accelerator of energetic electrons, creating a fluid like state of electrons. Electrons reach relativistic speeds under the fields of ultra intense lasers. Their relativisic mass increases reducing the effective plasma frequency. Thus, laser can penetrate even overdense plasmas at relativstic intensities. This effect is called relativistically induced transparency.

Since nonlinearity exists in high intensity laser plasma interaction, ponderomotive (PM) force should be taken into account. It is a nonlinear force that a charged particle experiences in an inhomogeneous oscillating electromagnetic field. It causes the particle to move towards the area of the weaker field strength rather than oscillating around an initial point as happens in a homogeneous field. Mathematically PM force can be understood as the force appearing after the time average of the force by oscillating field (which here is the electric field of laser). The time averaged kinetic energy of an oscillating electron is the ponderomotive potential.

Due to pondermotive force electrons in plasma are pushed out of the region of high intensity lasers. The dielectric constant of this electron cavity region changes in a manner that enhances the intensity of laser there. The cavity region also acts like a trap for radiation because the surrounding region is piled up with pushed out electrons making it overdense and inhibits laser propagation.

Development of lasers with short pulses and high power has been going on since the laser's invention in the 1950s. Currently, ultra short laser pulses emit ultrashort pulse of light that can turn on and off within femtoseconds and picoseconds. Many non linear optical phenomena have since been observed. One such observed example is self focusing and filamentation of ultra-short laser pulses.
When a femtosecond laser (ultrashort laser pulse) propagates, it self focuses with high intensity into a small volume of air, ionising some molecules. It keeps on propagating like a bullet in the same way for kilometres. A column of ionised molecules or plasma is left behind. The high-intensity core of the pulse is called the filament. Local defocusing brought on by the development of the plasma stops the collapse of the beam when diffraction, self-focusing, and plasma defocusing interact. The energy needed for ongoing ionisation depletes the pulse energy, causing the self-focusing to gradually deteriorate until it is finally defeated by diffraction and plasma defocusing, which causes the filament to burn out. The ionisation channel can stretch hundreds of metres during outdoor propagation and can be measured in units of a few meters. One of the main self-action effects in laser-plasma interaction is a phenomenon known as self-focusing. Since these phenomena are crucial for a number of high-power laser applications, including inertial confinement fusion, laser-electron acceleration, and harmonic production, it is essential to study these.

With the help of the plasma's nonlinear reaction, the laser beams can change the front medium during this process to make it more conducive to propagation. This phenomenon originated from electrostriction, Kerr, thermal, or ponderomotive effects.

A laser beam traversing a medium can modulate the refractive index of the medium as

$$N_i = N_0 + N_2 * I(r.t)$$

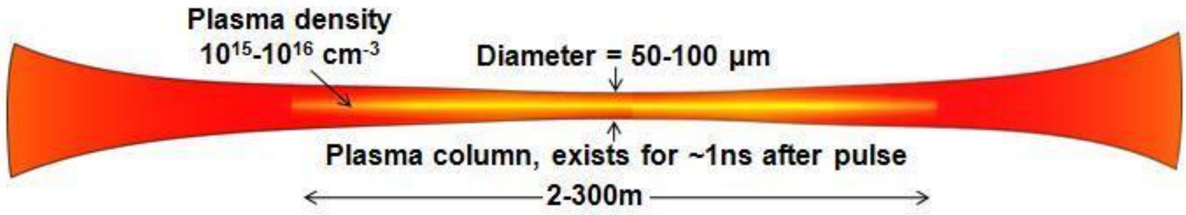
Where N_0, N_2 and I are the linear refractive index, second order refractive index and intensity of the propagating laser field, respectively. This intensity-dependent nonlinear component comes from two dominating contributions. One is the rise in the relativistic mass of electrons brought on by the electrons' quivering motion brought on by the laser beam's electric field. This causes a transverse gradient in the refractive index, which causes the beam's energy to be focused inward and reduces the beam's spot size. Another is the nonlinear electron density perturbations caused by the beam's ponderomotive force, which excites the electron plasma wave.

Relativistic self-focusing takes place on account of the quiver motion of the electrons due to a laser electric field while travelling at a speed comparable to the speed of light. The relativistic effect becomes dominant provided laser power exceeds its critical value,

$$P_{critical} = 17\left(\frac{w_p}{w_0}\right)^2 \text{ GW}$$

However, when the laser power is smaller than this critical power, diffraction of the beam dominates over relativistic self-focusing.

Once the pulse starts to self-focus, its intensity keeps increasing as photons move closer to the pulse centre. This unavoidably causes the medium to become multiphoton ionised, which results in the formation of an electron plasma in the pulse's centre. High electron densities reduce the medium's susceptibility during the ionising phase of the pulse, which locally lowers the index of refraction. Interesting energy dynamics within the pulse as it propagates result from the center's decrease in refractive index while the periphery maintains self-focus.



TYPICAL PICTURE OF A FILAMENT

The transverse beam profile is clearly divided into two sections after ionisation starts: the inner ionisation region and the outer energy reservoir. The inner ionisation area has the highest intensity and a diameter of 50–100 m. Although it has a significantly lower overall intensity and can reach a width of several millimetres, the outer region—dubbed the energy reservoir—often contains the majority of the total pulse energy. Photons in the central ionising zone of a healthy, uncontrolled filament undergo a decrease in refractive index and have a propensity to wander away from the beam centre. Peripheral photons that are outside of the ionisation region are still self-focusing and travelling in the direction of the beam centre. The balance between these two processes enables filament propagation to continue over time.

In underdense plasma, self-focusing has a fascinating quality. Extremely intense electric fields can be sustained by the plasma. Ion motion may not be taken into account for short laser pulses. In this event, the relativistic motion of electrons in the laser field determines the nonlinearity of light propagation.

The studies on electron plasma waves using high-energy electrons reveals that the electric field associated with electron plasma can reach a very large value in short distances[10-11]. Sharma et al.[12] investigated the transient evolution of laser pump in plasma under ponderomotive non linearity by numerically solving the coupled equations for laser beam and ion acoustic wave.

Their study finds that ion-acoustic spectra consists of spatial harmonics which modified with time and leads to the laser beam localization.

Sprangle et al.[13] followed the quasi-static approximation to solve the coupled equation for radiation vector potential field and plasma electrostatic potential field. This approximate one dimensional nonlinear model which describes the self-consistent interaction of laser and plasma. The three dimensional simulation studies using relativistic intensity of short laser pulse for the minimal under-dense plasma propagation reveals manifestation of mono-energetic electron beam bunches of energy about 300 MeV[14]. Further the study[15] on under-dense plasma using intense beam self-focusing describes the self-channelling of laser beam into the plasma provided the laser intensity is large enough to produce electron cavitation.

Brandi et al.[16] analyses the self-focusing of laser beam under quasi-steady state approximation and also derived the equations for laser beam and plasma oscillation dynamics under relativistic ponderomotive nonlinearity. Additionally, it was established that the plasma's inhomogeneity is crucial to the process of self-focusing. The analytical findings demonstrated that a change of a few per cent in the plasma's degree of homogeneity in its centre region might significantly alter the laser critical power value needed to trigger the self-focusing process.

The present study analyses the transient relativistic ponderomotive nonlinearity using computational method to solve the coupled equation. The generation of density harmonics and their effects on laser beam filamentation have also been investigated. In addition to the computation method a semi analytic model has also been presented to describe the nonlinear development of the laser beam.

The present study is very important as the transient evolution of electron density variation involves applications including harmonic generation of plasma oscillation in THz frequency range and filamentation of laser beam. This study is done at very high laser frequency compared to the electron plasma frequency for under dense plasma. The origin of nonlinear dynamics of plasma wave due to the relativistic ponderomotive nonlinear forces of laser beam has also been highlighted.

This paper is organized as follows: Sec. II discussed the coupled equations of laser and plasma oscillations in the plasma when the coupling arises via relativistic ponderomotive nonlinearity. Sec. III presented the numerical simulation results, Sec. IV presented transient self-focusing from semi analytical model and Sec. V consists of conclusion.

II. MODEL EQUATIONS FOR LASER AND PLASMA OSCILLATIONS

The atoms are ionised by the lasers in the time scale of femtoseconds or less. Such non linear atom-laser interaction cannot be solved by normal perturbation methods. Computational methods have to be applied to get solutions for such nonlinear dynamics.

The present analysis has been done for collision-less plasma coupled through the ponderomotive and relativistic nonlinearity. The dynamics of laser beam propagation associated with the plasma wave have been investigated as following:

A) EQUATION FOR THE LASER

The equation of continuity and momentum balance utilizing Maxwell's equations[17,18] have been used to study the propagation of linearly polarized intense laser beam of frequency w_0 along z-direction through the under-dense plasma having an equilibrium density n_0:

$$\frac{\partial n_j}{\partial t} + \nabla.(n_j \vec{v}_j) = 0 \quad (1)$$

$$\frac{\partial \left(m_j \vec{v}_j\right)}{\partial t} + \vec{v}_j.\nabla\left(m_j \vec{v}_j\right) = q_j \left\{ \vec{E} + \frac{(\vec{v}_j \times \vec{B})}{c} \right\} \quad (2)$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \quad (4)$$

where, j symbolizes plasma species (e for electron and i for ion) and q_j, m_j, v_j, n_j and E symbolize the charge, mass, velocity, number density and energy of the plasma species respectively.

The x-component of wave equation is obtained as following :

$$\nabla^2 E_x - (\nabla\left(\nabla.\vec{E}\right))_x = \frac{4\pi}{c^2}\frac{\partial J_x}{\partial t} + \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \quad (5)$$

If ratio of spatial variation of nonlinear plasma dielectric constant with the non-linear plasma dielectric constant is unity then the term $(\nabla(\nabla.(\vec{E})))_x$ [19] can be neglected and the solution of equation (5) results as follows[17,19]:

$$E_x = A\left(x,z,t\right)\exp\left\{-i\left(\omega_0 t - k_0 z\right)\right\} \quad (6)$$

where, $A(x,z,t)$ is the wave amplitude of slow space and time dependence, $k_0 = \frac{w_0}{c}[1 - \frac{w^2{}_{p0}}{\gamma w^2{}_0}]^{\frac{1}{2}}$,

$$w^2{}_{p0} = 4\pi n_0 e^2/m_{e0} \,,$$

m_{e0} is the electron rest mass,

$\gamma = (1 + a^2/2)^{\frac{1}{2}}$ is the relativistic factor and

$$a = \frac{e\ |A\,|}{m_{e0}\omega_0 c}$$

In terms of current density drift velocity above equation is reduced to Eq. (7):

$$J_x = -\frac{n_e e^2}{m_e i\omega_0}\left[E_x - \frac{i}{\omega_0}\frac{\partial A}{\partial t}\mathrm{e}^{-i(\omega_0 t - k_0 z)}\right] \tag{7}$$

where, $J_x = -e n_e v_e$ is current density,

$n_e = n_0 + n_1$ is electron density and

$\vec{v}_e = e\vec{E}/m_e i w_0$ is drift velocity.

In WKB approximation[19,23] and utilizing Eqs. (6) and (7) in Eq. (5), one gets the following expression:

$$2ik_0\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} + \frac{2i\omega_0}{c^2}\frac{\partial A}{\partial t} = \frac{\omega_{p0}^2}{c^2\gamma}\frac{n_1}{n_0}A \tag{8}$$

B) DYNAMICS OF PLASMA WAVE

Under the relativistic ponderomotive force[16] $\vec{F}_p = -m_{e0}c^2\nabla(\gamma - 1)$ generated by laser beam wavefront having nonuniform amplitude, electrons move away from the maximum field amplitude region. If the laser pulse duration is smaller than the plasma ion period then plasma ions do not respond in phase with the laser pulse resulting a charge imbalance between the ions and electrons which in turn produces a space charge field $(\vec{E}_p)$. The expression for electron density is obtained by differentiating equation of continuity (Eq. 1) and using Eq. (2). It is given as following:

$$\frac{\partial^2 n_e}{\partial t^2} - \frac{e\nabla.\left(n_e \vec{E}\right)}{m_{e0}\gamma} = \nabla.\left\{\frac{\vec{v}_e.\nabla\left(\gamma \vec{v}_e\right)}{\gamma} + \frac{e\vec{v}_e \times \vec{B}}{m_{e0}\gamma c}\right\} n_e - \nabla.\left\{-\frac{n_e}{\gamma}\frac{\partial \gamma}{\partial t} + \frac{\partial n_e}{\partial t}\right\}\vec{v}_e \tag{9}$$

Linearization of Eq. (9) under the condition ($n_e = n_0 + n_1$, where $n_0 >> n_1$) with Poisson relation ($\nabla.\vec{E_p} = -4\pi n_1 e$) , the nonlinear dynamical wave equation of plasma wave equation is given as follows:

$$\left(\frac{\partial^2}{\partial t^2} - \frac{v_{th}^2}{\gamma}\frac{\partial^2}{\partial x^2} + \frac{\omega_{p0}^2}{\gamma}\right)n_1 = -\frac{c^2 n_0}{4\gamma^2}\left[\left(\frac{e}{m_{e0}\omega_0 c}\right)^2\left(\frac{\partial^2 |A|^2}{\partial x^2} + \frac{\partial^2 |A|^2}{\partial z^2}\right) + \left(\frac{e}{m_{e0}\omega_0 c}\right)^4\left(\left(\frac{\partial |A|^2}{\partial x}\right)^2 + \left(\frac{\partial |A|^2}{\partial z}\right)^2\right)\right]$$

(10)

In dimensionless form Eq. (8) and Eq. (10) can be written as given below:

$$i\frac{\partial A'}{\partial t'} + i\frac{\partial A'}{\partial z'} + \frac{\partial^2 A'}{\partial x'^2} = c_4 n' A' \tag{11}$$

$$\frac{\partial^2 n'}{\partial t'^2} - c_5\frac{\partial^2 n'}{\partial x'^2} + c_1 n' = -c_2\left[\frac{\partial^2 |A'|^2}{\partial x'^2} + \left(\frac{\partial |A'|^2}{\partial x'}\right)^2\right] - c_3\left[\frac{\partial^2 |A'|^2}{\partial z'^2} + \left(\frac{\partial |A'|^2}{\partial z'}\right)^2\right] \tag{12}$$

here,

$c_1 = \frac{\omega_{p0}^2 t_n^2}{\gamma}$, $c_2 = \frac{c^2 t_n^2 n_0}{4\gamma^2 x_n^2 n_n}$, $c_3 = \frac{c^2 t_n^2 n_0}{4\gamma^2 z_n^2 n_n}$, $c_4 = \frac{\xi}{\gamma}$ and $c_5 = \frac{v_{th}^2 t_n^2}{\gamma x_n^2}$.

The normalized quantities are given as following:

$t_n = \frac{2\eta\,\omega_0}{c^2k_0^2}$, $x_n = \frac{(\eta)^{0.5}}{k_0}$, $z_n = \frac{\eta}{k_0}$, $n_n = \frac{c^2k_0^2}{\eta\,\omega_{p0}^2}n_0$ and $A_n = \frac{\varsigma m_{e0}\omega_0 c}{e}$. where, $\eta = 1200$, $\xi = 20$

and $\varsigma = 10$.

The normalization parameter z_n and n_n are related with the wave number k_0 of laser beam and background number density n_0 of plasma.

Eq. (11) and Eq. (12) were solved numerically to explore the impact of plasma wave on the laser beam localization process and plasma wave hormonic generation.

III. NUMERICAL SIMULATION AND RESULTS

The initial condition of the simulation is considered as

$$A'(x,z,0) = A_0\left(1+\beta\cos\left(\alpha_x x'\right)\right)\left(1+\beta\cos\left(\alpha_z z'\right)\right) \tag{13a}$$

$$n'(x,z,0) = -n_{10}\left|A'(x,z,0)\right|^2 \tag{13b}$$

For the initiation of numerical computation, the amplitude A_0 of laser beam and electron plasma density fluctuation n_{10} were taken as unity (=1). The perturbation magnitude (β), perturbation wave numbers (α_x, and α_z) were taken equal to 0.1. The equations are numerically solved by considering the 256x256 grid points in a spatial periodic domain step size $\left(\frac{2\pi}{\alpha_x}\right)x\left(\frac{2\pi}{\alpha_z}\right)$. The space integration and time evolution have been obtained by using pseudo-spectral method and finite difference method respectively. The predictor corrector scheme[20] is utilized for these evaluations. Time step width was chosen as 5 micro seconds. The reliability of the presented algorithm has been investigated by first using it for nonlinear Schrödinger wave equation to obtain constant plasmon number. Thereafter, this algorithm is altered to solve the equations presented in this study.

The parameters used in simulation are:

$\omega_0 = 2.35\times10^{15}\, rad/\sec$, $\omega_{p0} = 0.001\omega_0$,

n_0= 1.74 x 10^{15} cm^{-3} (under-dense region),

a=1.0 and

$k_{0z} = 7.84\times10^4\, cm^{-1}$.

The normalizing parameters are x_n=4.4x10^{-4}cm, z_n= 3.05x10^{-2} cm, t_n= 1.02 picoseconds and E_n=1.34x10^7 stat V/cm. Figs. 1(a)-1(c) show the localized structures of field intensity, $|A'|^2$ of laser beam in x-z plane at distinct time.

The nonlinear force from the laser pump causes the electron plasma wave to perturb the background electron density. Due to perturbation, the variation of density reflected in the form of modes/harmonics which in turn forms nonlinear coupling between plasma wave and laser beam. The nonlinear coupling produces filamentation in laser beam and hence the localized structures. Fig. 1 (a) presents the initiation of the localized structure formation. As time passes the, localized structure becomes more and more complex as shown in Fig. 1 (b) – (c).

The variation of surface electron density in x-z plane at normalized time $t'=5$ is shown in Fig. 2. One can observe the density depletion in some zones of the x-z plane. The density cavities formations are time dependent and forms at different time related to the coherent electric field structure.

The frequency spectrum of electron plasma shown in Fig. 3 infers the excitation of plasma waves in THz range. This behaviour is observe because different density oscillations produced corresponding to different wave vectors ($k_x = \alpha_x, 2\alpha_x, 3\alpha_x$ upto $64\alpha_x$) and peaks arises in THz frequency region. The similar behaviour i.e. generation of acoustic waves in THz region have also been observed experimentally[21].

The response of normalized density $|n|$ mode with wave vector (k_x) along x-axis (when $k_z = 0$) is shown in Figs. 4 (a) and 4(b). It is observed that as time lapses the higher order density modes are generated. The higher order harmonics in density are generated due to the application of nonlinear force by intense laser beam[22].

With constraint $k_z = \alpha_z = 0$, Eq. (13a) and Eq. 13(b) takes the form given as below

$$A'(x',0) = 1 + \beta\cos(\alpha_x x') \tag{14}$$

$$n'(x',0) = -\left(1 + \beta\cos(\alpha_x x')\right)^2 = -\left(1 + \beta^2\cos^2(\alpha_x x') + 2\beta\cos(\alpha_x x')\right) \tag{15}$$

Eq. (15) shows the generation of density harmonics corresponding to α_x and 2α_x. Combining equation (14), (15) and (11) one can observe the generation of density harmonics corresponding to α_x, 2α_x and 3α_x as given in Eq. (16).

$$n'A' = -\left(1 + \beta\cos(\alpha_x x')\right)\left(1 + \beta^2\cos^2(\alpha_x x') + 2\beta\cos(\alpha_x x')\right) \tag{16}$$

Following the same procedure the higher order density harmonics corresponding to $k_x = \alpha_x,\ 2\alpha_x,\ 3\alpha_x$ upto $64\alpha_x$ in the spectrum of $|n|$ can be obtained, see Fig. (4) at the normalized time, t' =3 and 23.

IV. TRANSIENT SELF-FOCUSING: A SEMI ANALYTICAL APPROACH

A semi analytical model based on the findings of the computational study has been presented in this section. This approach describes the role of nonlinear progression of laser beam as the localize structure and variable number density[12,22]. For spatial variation only (neglecting the time variation), the wave equation under the WKB approximation for wave having rapid phase variation is given as follows:

$$2ik_0 \frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} - \frac{k_0^2}{\eta\gamma}\left(\frac{n_e}{n_n}\right) A = 0 \tag{17}$$

The values of normalizing electron density (n_n) and time dependent terms $\left(\frac{n_e}{n_n}\right)$ are found from computational results. Expansion of the time dependent terms gives:

$$\frac{n_e}{n_n} = N + \sum_{j=1}^{64} n_j \left(\cos\left(j\alpha_x x'\right)\right) = N + \sum_{j=1}^{64} n_j \left(1 - \frac{j^2 x'^2 \alpha_x^2}{2} + ...\right) \tag{18}$$

N and n_j are the plasmon number and number density of harmonics respectively, these terms are obtained from the computational results. Solution of Eq. (17) is given as follows[23]:

$$A = A_0\left(x, z\right) e^{ik_0 s(x,z)} \tag{19}$$

Substituting Eq. (18), (19) in Eq. (17) and equating the real and imaginary part one gets the following equations:

$$-2k_0^2 A_0 \frac{\partial S}{\partial z} + \frac{\partial^2 A_0}{\partial x^2} - k_0^2 A_0 \left(\frac{\partial S}{\partial x}\right)^2 - \frac{k_0^2}{\xi \gamma} A_0 \left(N + \sum_{j=1}^{64} n_j \left(1 - \frac{j^2 x'^2 \alpha_x^2}{2} + ... \right) \right) = 0 \tag{20}$$

and

$$2\frac{\partial A_0}{\partial z} + 2\frac{\partial A_0}{\partial x}\frac{\partial S}{\partial x} + A_0 \frac{\partial^2 S}{\partial x^2} = 0 \tag{21}$$

For an eikonal and Gaussian beam of initial beam width r_0, A_0 and S with E_{00} initial laser beam amplitude are given as below:

$$A_0 = \frac{E_{00}}{f^{1/2}} \exp\left(-\frac{x^2}{2r_0^2 f^2} \right) \tag{22}$$

and $$S = \frac{x^2}{2f}\frac{\partial f}{\partial z} \tag{23}$$

On substitution of above equations in Eq. (20) and equating the coefficients of x^2 in both sides one can obtain the beam width parameter f (under paraxial ray approximation) as given below:

$$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3} - \frac{1}{\xi\gamma}\left(\frac{r_0^2\omega_0^2}{c^2}\right)\frac{1}{f^2}\left(\frac{E_{00}}{E_n}\right)^2 - \frac{1}{\xi\gamma}\left(\frac{r_0^4\omega_0^2}{c^2}\right)\frac{f\alpha_x^2}{2}\sum_{j=1}^{64} n_j j^2 \tag{24}$$

Eq. (24) has been solved using fourth order Runge Kutta method under the initial boundary conditions $f = 1,\ df/d\xi = 0$ at $\xi = 0$. The wave propagate without divergence/convergence if the diffraction term (first term in R.H.S. of Eq. (24)) is balanced by nonlinear terms (2^{nd} and 3^{rd} terms of in R. H. S. of Eq. (24)). The transient progression effects in laser beam are observed due the 3^{rd} term in R. H. S. of Eq. (24). As laser beam propagates through the plasma, due to nonlinearity laser beam attains self-focussing. For longer time, nonlinearity decreases and diffraction term in Eq. (24) dominates over the nonlinear terms and laser beam defocusses. Imbalance between diffraction term and nonlinear term produces the oscillatory self-focussing of the laser beam. This can be clearly observed from Figs. 5(a) and 5(b) where laser beam localization at two different times has been shown. Initially the impact of density harmonics via nonlinearity is trivial and no focussing was observed. As time passes evolution of density harmonics contributes and one observes the laser beam focussing.

V. CONCLUSION

The computational technique has been used to study the transient evolution of laser beam for relativistic ponderomotive nonlinear force in under-dense plasma. The time varying localization process of plasma has been observed which was more complex at longer times. In plasma wave frequency spectra, peaks have been observed in THz region. In addition to the filamentary density depletions, substantial density cavities has also been observed. Periodic perturbation produced the density harmonics which affects the localization of laser beam. A semi analytical model has been developed using the computational results to study the time varying self-focussing phenomenon of laser beam. The semi-analytical model predicts that time varying self-focussing of laser beam depends on the values of density of harmonics. The present model is applicable in experimental and theoretical studies involving transient localization of laser beam in plasma.

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FIGURE CAPTIONS :

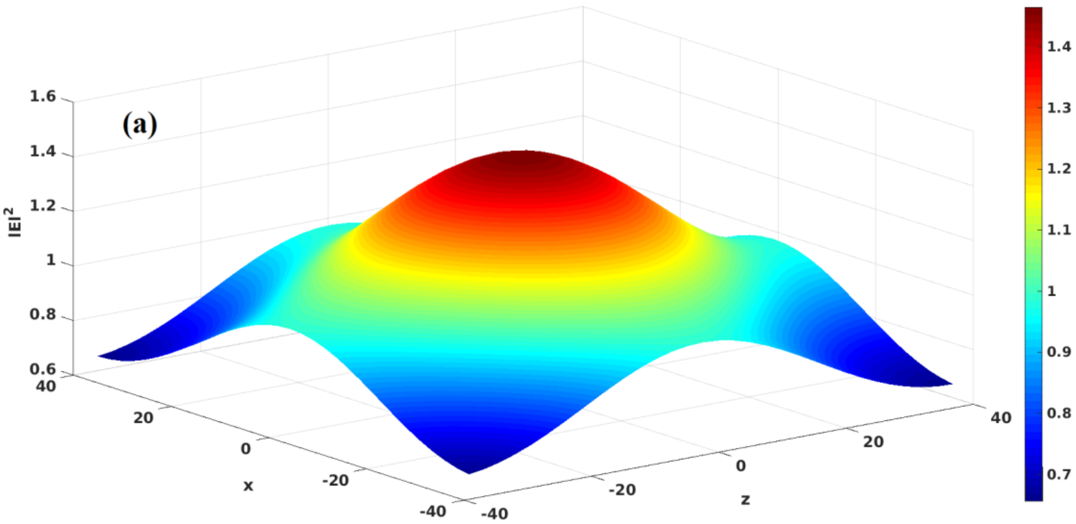
FIG 1: Normalized intensity profile of laser beam in x-z plane at distinct time (a) $t'=0$ (b) $t'=8$ and (c) $t'=16$.

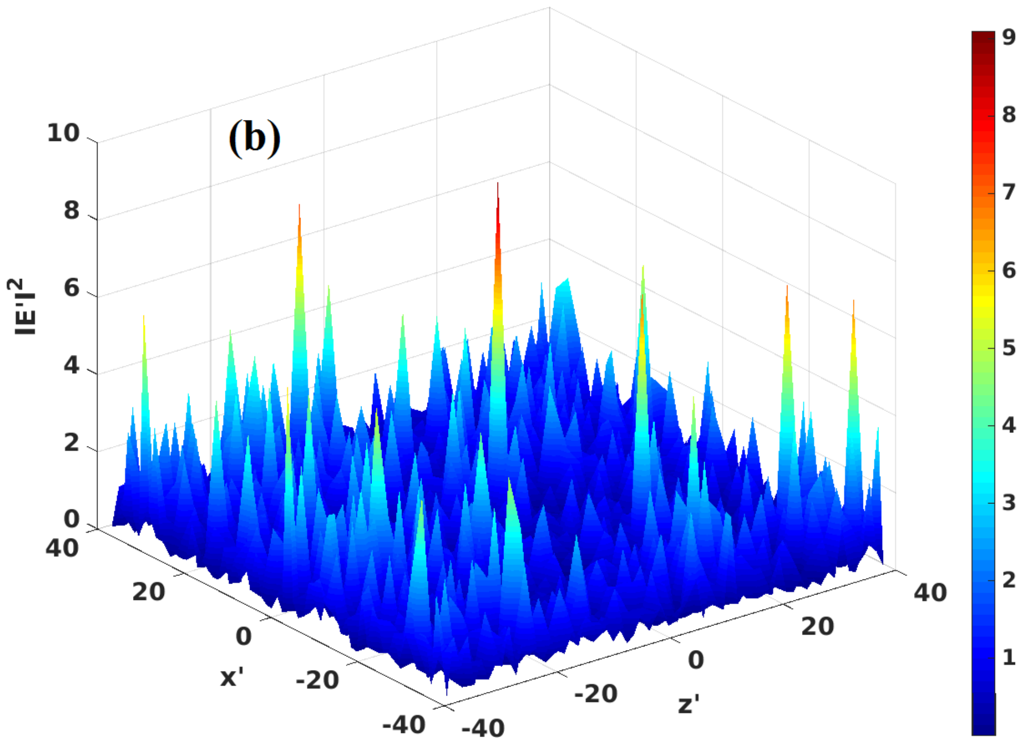
FIG 2: Plot of normalized electron density in x-z plane at normalized time, $t'=5$.

FIG 3: Plot of Normalized frequency spectrum of the plasma density oscillations.

FIG 4: Variation of normalized electron density $|n|$ versus k_x (at fixed k_z) for normalized time (a) $t' = 3$ and (b) $t' = 23$.

FIG 5: Normalized intensity profile of laser beam in paraxial regime obtained by semi-analytically model in the x-z plane at distinct time (a) $t' = 2$ and (b) $t' = 23$.





(b)
|E'|^2
x'
z'

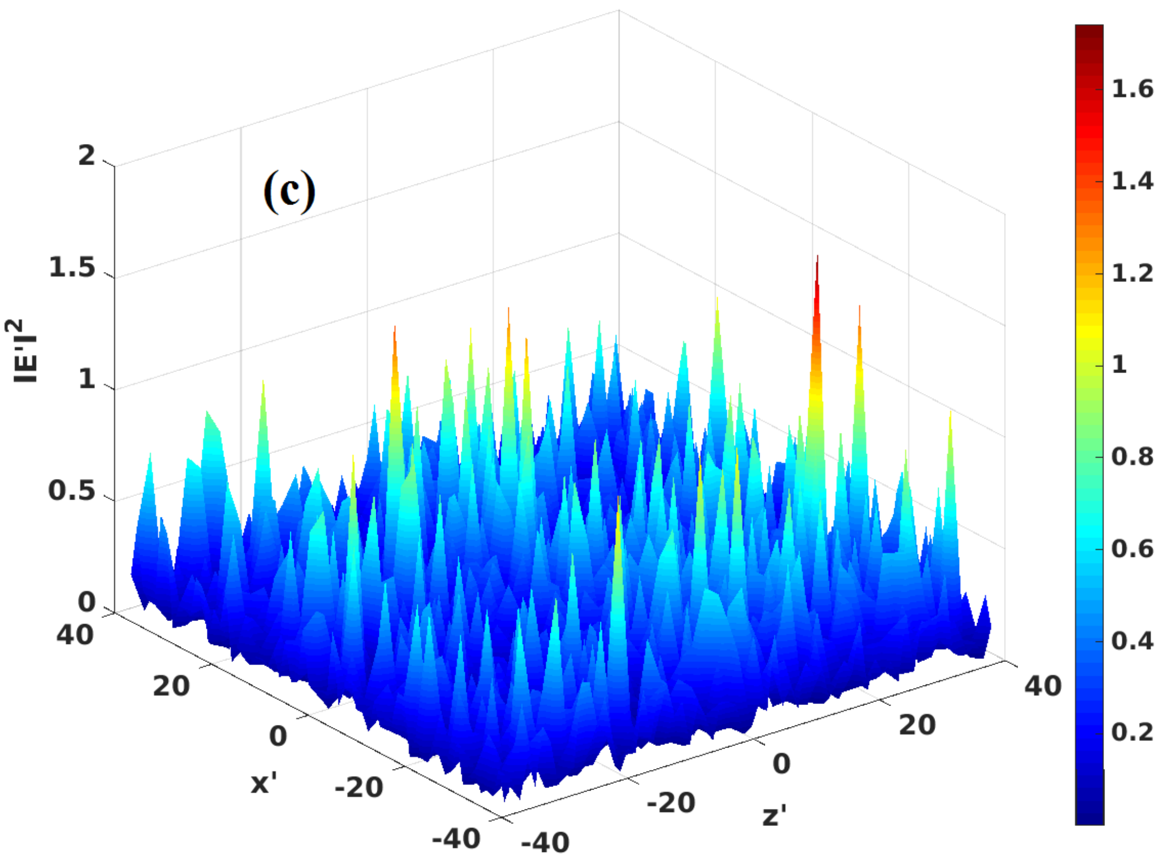


FIG 1: Normalized intensity profile of laser beam in x-z plane at distinct time (a) $t'=0$ **(b)** $t'=4$ and **(c)** $t'=7$**.**

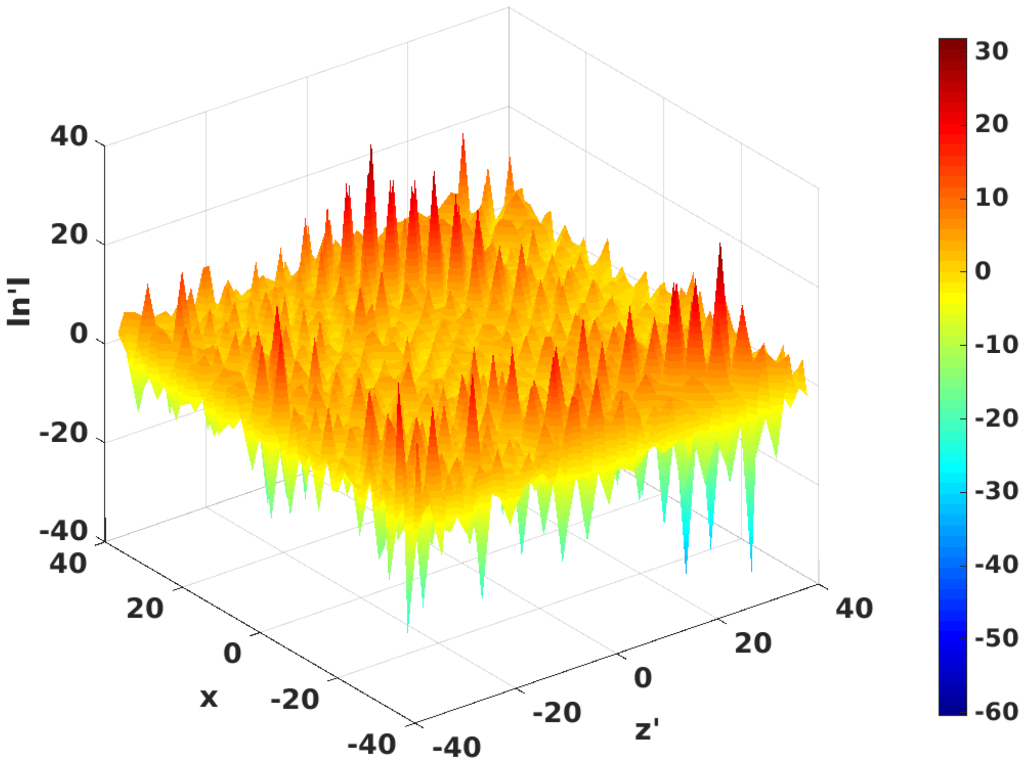


FIG 2: Plot of normalized electron density in x-z plane at normalized time $t' = 5$.

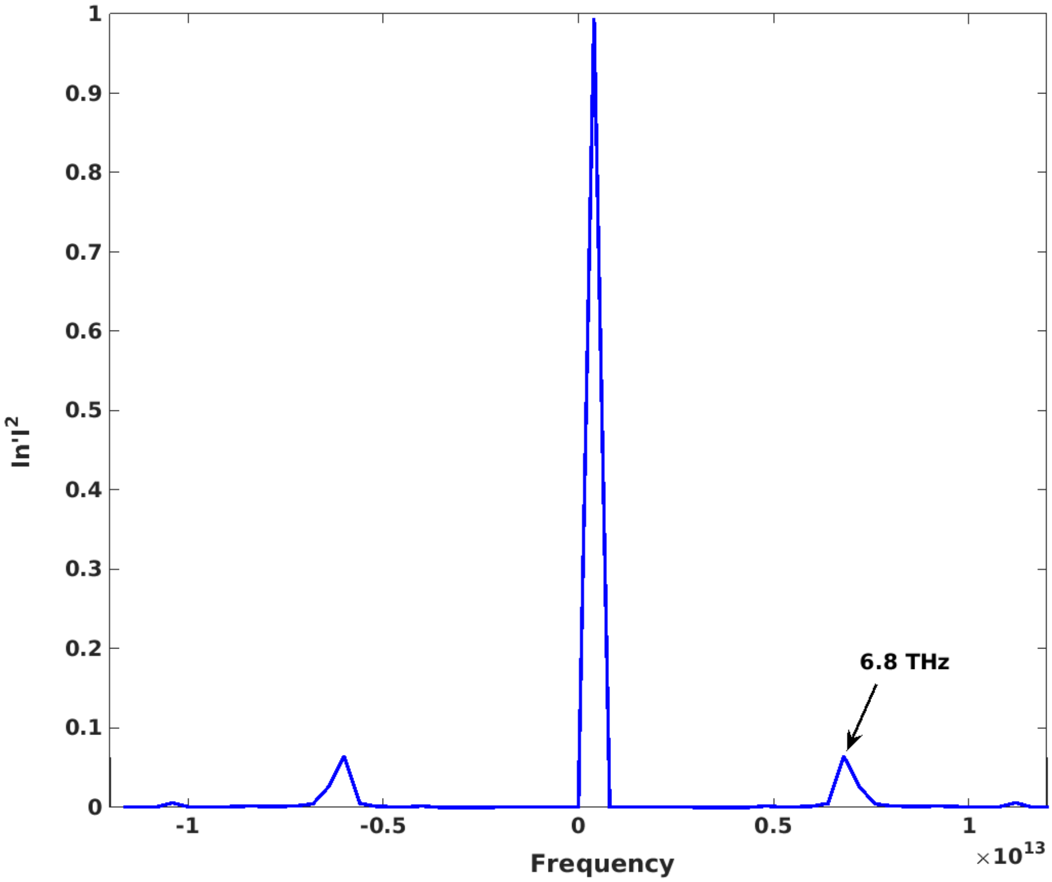
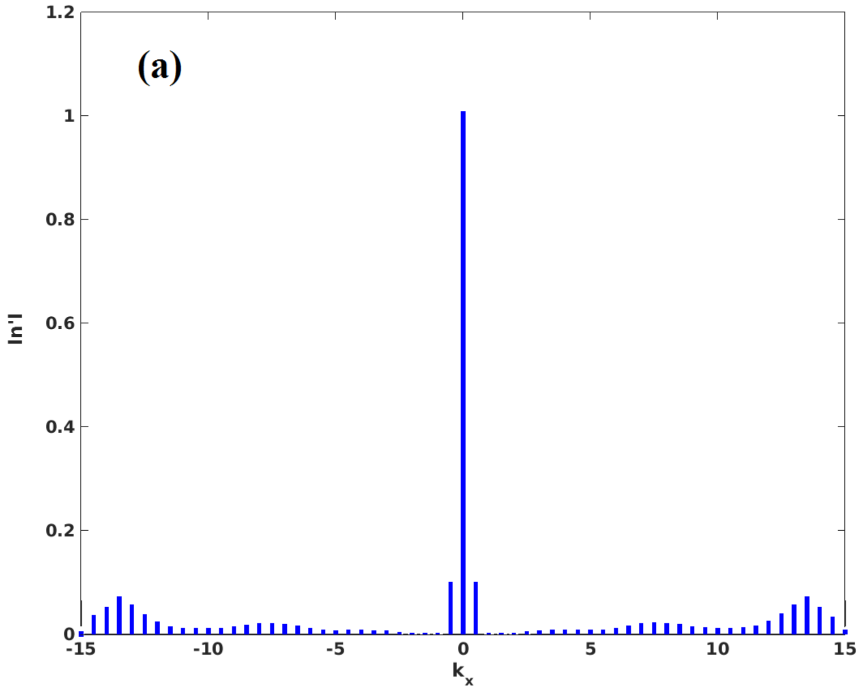


FIG 3: Plot of normalized frequency spectrum of the plasma density oscillations.


(a)
1.2
1
0.8
0.6
0.4
0.2
0
In'I
-15
-10
-5
0
5
10
15
k_x

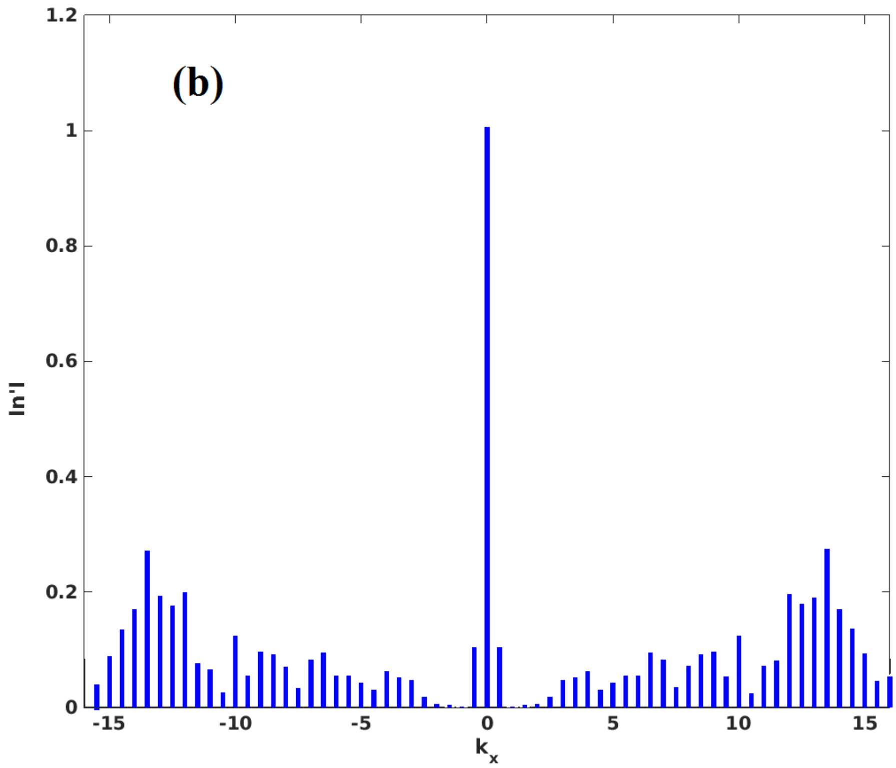
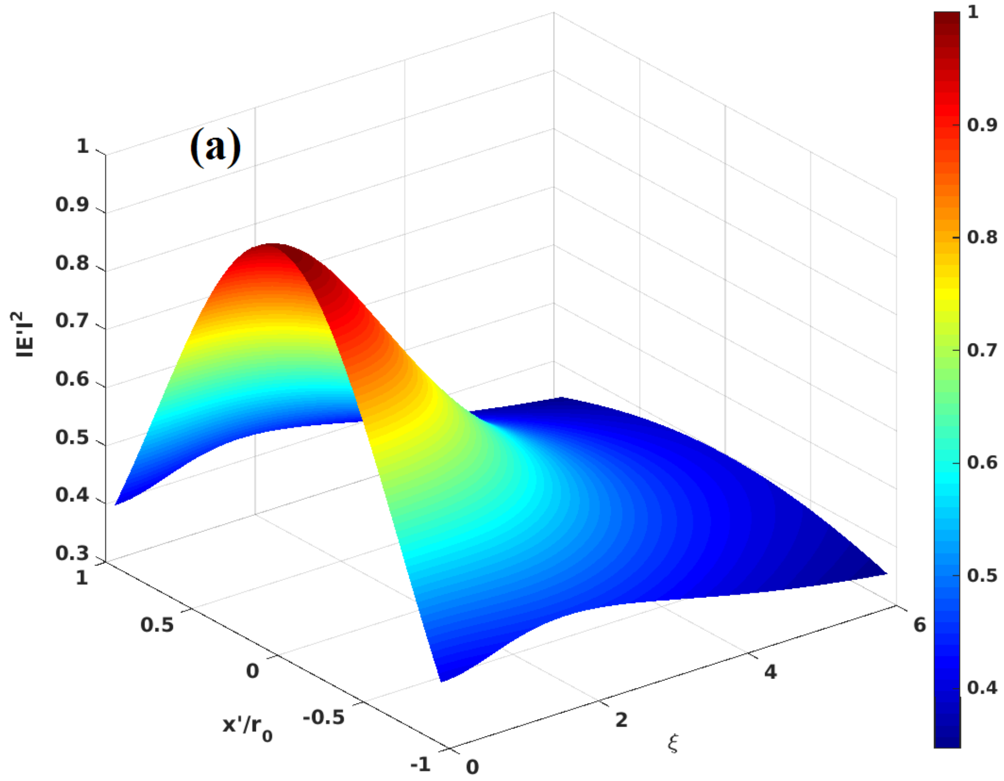

(b)
1.2
1
0.8
0.6
0.4
0.2
0
In'I
-15
-10
-5
0
5
10
15
k_x

FIG 4: Variation of normalized electron density $|n|$ versus k_x (at fixed k_z) for normalized time (a) $t' = 1$ and (b) $t' = 8$.



(a)
IE'I^2
x'/r$_0$
ξ

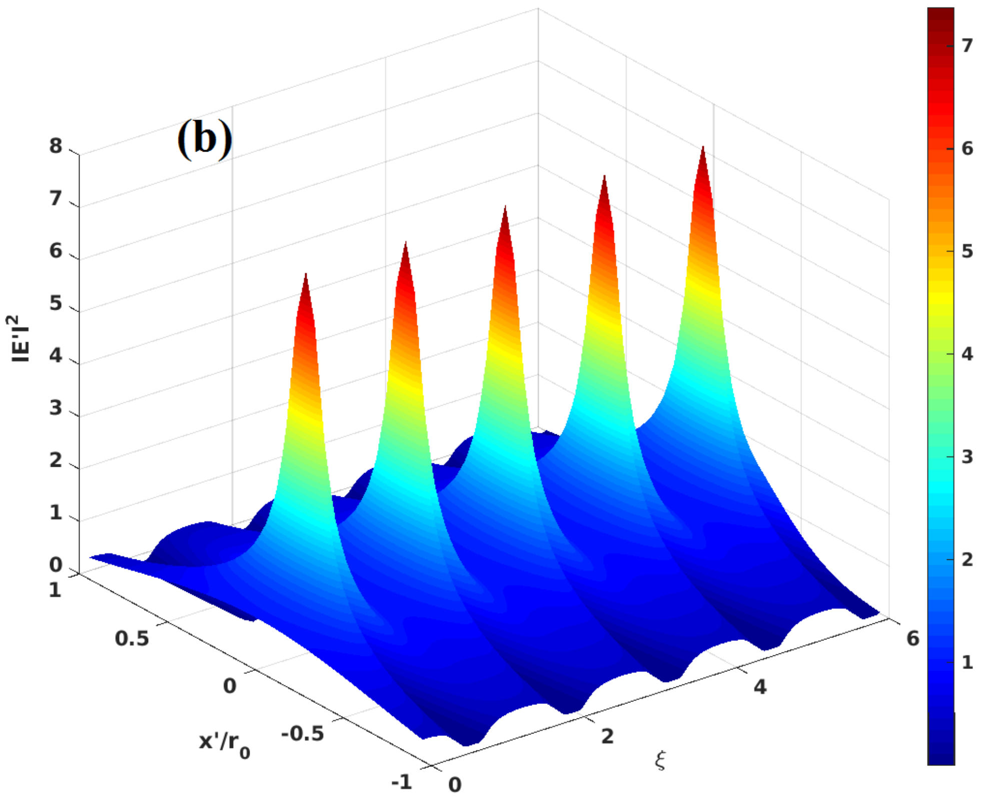


FIG 5: N**ormalized intensity profile of laser beam in paraxial regime obtained by semi-analytically model in the x-z plane at distinct time (a)** $t'=1$ **and (b)** $t'=8$.